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Integrated framework for preference modeling and robustness analysis for outranking-based multiple criteria sorting with ELECTRE and PROMETHEE

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A B S T R A C T

We introduce an integrated framework for preference modeling and robustness analysis in outranking-based multiple criteria sorting. The preference information supplied by the Decision Maker (DM) is composed of parts of desired recommendation and imprecise requirements that the delivered outcomes should satisfy. Its allowed forms are: imprecise assignment examples, desired class cardinalities, and assignment-based pairwise comparisons. The exploitation of all instances of the outranking model compatible with these preferences results in three types of outcomes. These raise robustness concerns in terms of the stability of a suggested assignment for each alternative, an observed size of each class, and a comparison between recommendation delivered for pairs of alternatives. The correspondence between different types of inputs and outputs facilitates the dialogue with the DM and enhances her/his confidence in the suggested recommendation. While referring only to a semantic meaning of an outranking relation, we present the procedures for translating the preference information into parameters of compatible outranking model instances and for deriving various kinds of sorting results. Then, we implement this general framework in the context of outranking models specific for ELECTRE and PROMETHEE. Finally, we show how preference modeling and robustness analysis can be performed and greatly simplified with a set of preference model instances providing precise assignments for the alternatives. The framework proposed for this case is based on Mixed-Integer Linear Programming (MILP), being independent of the underlying model and method. Application of the approaches is demonstrated on the case of classifying Polish research units into four quality classes.

1. Introduction

In multiple criteria sorting problems alternatives need to be assigned to a finite and completely ordered set of decision classes. Each class is pre-defined to receive alternatives that will be processed in the same way. The semantic definition of classes depends on the particular sorting domain. In medicine, one may assess the severity of breast cancer patients according to several symptoms. In agriculture, parcels may be grouped into different risk levels related to protecting the reproduction habitat of fishes [36]. Risk categories may be also considered in nanotechnology to evaluate the materials

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based on their physico-chemical characteristics and expected environmental impact [52]. Further, world locations may be assigned to climate categories ranging from unfavourable to ideal to tourism [37], whereas users or documents are often assigned to the sorted groups in recommender systems [38,54]. Finally, one may assess project activities into classes of managerial practices: the highest class requires the greatest managerial attention, while the supervision of the lowest class activities could be delegated to a subordinate [41].

1.1. Review of outranking-based assignment rules and their real-world applications

Over the last twenty years a large body of research in the field of outranking-based sorting methods appeared. Although using the same preference model, these approaches propose various ways for defining decision classes and employ different assignment rules. In particular, we may consider one or several boundary profiles to define each class. In this case, an alternative may be assigned to a particular class if it outranks at least one lower boundary profile, while not being preferred to any upper boundary profile. This approach has been implemented, e.g., in the ELECTRE TRI-B [57] and FPP (Filtering by strict preference) [47] methods. It has been found appropriate for the context of real-world problems in, e.g., economy [13], risk zoning [40] and assessment [7], environmental protection [11], electricity market [39], stock portfolio selection [55,56], water resources management [48], land-use suitability assessment [4,26], evaluation of airport performance [46], and recommender systems [54].

A different outranking-based sorting approach assumes that each class is defined with one or several characteristic profiles or prototypes. An underlying assignment rule classifies an alternative in some class if it is judged similar, indifferent or equivalent to at least one prototype of this class [5], or if it is considered better than a characteristic profile of the previous class and worse than a profile defined for the subsequent class [34]. This procedure has been implemented, e.g., in the ELECTRE: TRI-C [1], TRI-nC [2], TRI-rC [34], and SORT [24] methods, and already applied in real-world problems in agriculture [36], medical diagnosis [18], project management [41], and tourism [37].

In this paper, we refer to yet another sorting rule which derives from the semantic meaning of an outranking relation. Precisely, if alternative *a* outranks alternative *b*, the class of *a* should be at least as good as the class of *b*. This intuitive and relatively strong rule has been advocated in [49] and [35]. At the same time, these two approaches implement a new direction in outranking-based multiple criteria sorting with respect to a way of defining the decision classes. Instead of requiring the Decision Maker (DM) to explicitly provide either class limits or characteristic profiles, (s)he is asked to specify only some assignment examples, i.e., to sort existing alternatives which are relatively well-known to her/him, or for which it is relatively easy to specify a desired sorting recommendation. For example, the DM may require that "*a* should be assigned to the best class, while *b* goes to the worst one". The role of such assignment examples is to form a set of benchmarks within the set of existing alternatives defining decision classes for the possible assignment of the remaining alternatives.

1.2. Motivation: what is missing in the existing outranking-based sorting methods?

In an outranking-based sorting model, the role played by each criterion needs to be characterized by several important parameters. The intra-criteria model parameters, such as indifference and preference thresholds, can be provided by the DM without much cognitive effort in real-world applications [50], but inter-criteria parameters are harder to elicit [34]. Consequently, the main research effort has been related to an indirect inference of the values for the criteria weights. In particular, [49] and [35] consider this problem in the context of an ELECTRE-based model. Then, for each alternative they provide its possible assignment, i.e., a set of classes the alternative can be assigned to by at least one compatible outranking model instance.

The vast majority of disaggregation outranking methods employ just a single preference modeling approach based on the assignment examples as well as a unique way of conducting robustness analysis which results in the possible assignments. However, one can indicate some other types of indirect preference information that have not received due attention in outranking-based methods, despite people commonly formulate such requirements in practical situations. Such holistic judgments involve assignment-based pairwise comparisons (e.g., "there is a difference of at least one class between *a* and *b*") [27] and desired class cardinalities (e.g., "at least one fourth of the alternatives should be assigned to the best class") [30,42].

Incorporating a number of preference modeling approaches into a single modeling approach which captures preference information given in different indirect forms is beneficial for several important reasons [27]. First, increasing the flexibility of the interaction, it significantly facilitates the dialogue with the DM when constructing an internal mathematical preference model representing elements of her/his value system. Second, the delivered recommendation better corresponds to the DM's value system and characteristics of a particular multiple criteria problem including requirements imposed by a decision context. Finally, since the space of compatible preference model instances defined by several pieces of preference information is more constrained, the delivered recommendation is more precise, decisive, and robust.

With different perspectives that can be referred at the input of the sorting method, it is desirable to enrich robustness analysis by deriving different sorting results so that their types correspond to admitted preference information. With focus on a single alternative, one may consider the necessary assignments along with the possible ones. The necessary assignment is valid for all compatible preference model instances, reflecting certainty with respect to the recommendation. When referring to the pairs of alternatives, we can verify the stability of assignment-based preference relation which holds if one of them is assigned to a class at least as good as the other [22,33]. Finally, with focus on decision classes, we may check what are the observed minimal and maximal class cardinalities. Overall, these outputs can be seen as elements of method's responses which materialize the consequences of DM's preferences on the set of alternatives, all pairs of alternatives, and decision classes. Analyzing a diversity of robust results is beneficial for enhancing both the DM's confidence in the suggested recommendation as well as organization of the interactive preference elicitation process.

1.3. Main contribution of the paper

The aim of this paper is three-fold. First, we propose an integrated framework for preference modeling and robustness analysis in outranking-based multiple criteria sorting. We ensure the input-output correspondence with respect to three perspectives for multiple criteria sorting concerning:

- individual alternatives – input: possibly imprecise assignment examples, and output: necessary and possible assignments;
- decision classes – input: desired class cardinalities, and output: observed size of each class;
- pairs of alternatives – input: assignment-based pairwise comparisons, and output: necessary assignment-based preference relation.

We discuss how to translate different types of preference information into parameters of compatible outranking model instances and how to derive various kinds of sorting results, referring only to a semantic meaning of an outranking relation. In this perspective, our proposal can be seen as an outranking-based counterpart of the recent value-based sorting framework [27]. In any case, our aim is to provide a general approach employing an intuitive assignment rule that can be subsequently implemented in terms of any outranking model.

In this perspective, the second aim of the paper consists in discussing two specific implementations of the general framework. On the one hand, we refer to an outranking model defined in the spirit of ELECTRE methods. In this way, we extend the approaches presented in [49] and [35] in several ways. These are based on precise assignment examples and deriving the possible assignments only. We admit imprecision in the exemplary sorting decisions and provide the necessary assignments. Moreover, we additionally account for two sorting perspectives referring to pairs of alternatives and decision classes. The former is materialized with the assignment-based pairwise comparisons and preference relations, whereas the latter incorporates desired and extreme observed class cardinalities.

On the other hand, we consider an outranking model which is characteristic for PROMETHEE methods. The family of existing PROMETHEE-based sorting approaches includes PROMSORT [3], PROMETHEE TRI [16], and FlowSort [25,44,45]. These methods employ central (characteristic) or boundary profiles to define the classes. The sole preference disaggregation method incorporating the pairwise comparisons concepts from PROMETHEE is PairClas [14]. It asks the DM to provide assignment examples, and derives a single compatible outranking model instance consisting of criteria weights, class thresholds, and comprehensive net outranking flows of alternatives. For working out the precise assignment for the alternative, it incorporates a threshold-based sorting procedure (the same sorting rule is used in [23]). The PROMETHEE-based sorting method proposed in this paper differs from PairClas by providing a variety of preference modeling techniques, investigating robustness of the delivered recommendation rather than arbitrarily selecting a single preference model instance, and employing a different sorting procedure, which avoids imposing the class thresholds on the scale of net outranking flows.

Finally, the third contribution of this paper consists in showing how the complexity of mathematical preference modeling and robustness analysis can be reduced when considering a set of preference model instances providing precise assignments for the alternatives. Even though for illustrative purpose we consider the settings of PairClas and (value-based) UTADIS [10,59], the proposed framework can be used with any outranking- or value-based method as long as it delivers precise assignments with a single preference model instance. Procedures for translating the preference information into parameters of compatible preference model instances and for robustness analysis are based on Mixed-Integer Linear Programming (MILP).

The paper is organized as follows. In Section 2, we recall the outranking preference models which have been used in the proposed methods. Section 3 is devoted to preference modeling for different types of preference information supplied by the DM. Section 4 presents the procedures for robustness analysis in the context of the set of compatible outranking model instances. Section 5 is devoted to a "no jump" property in the possible assignments. Section 6 demonstrates the use of introduced approaches. In Section 7, we focus on models whose instances deliver precise assignments for the alternatives. The last section concludes.

2. The outranking preference model: definitions and notation

We use the following notation:

- $A = \{a, b, \ldots\}$ – a finite set of n decision alternatives.
- $F = \{g_1, \ldots, g_j, \ldots, g_m\}$ – a consistent family of m criteria, $g_j : A \to \mathbb{R}$; we assume, without loss of generality, that all criteria are maximized.
- $C_1, \ldots, C_h, \ldots, C_t$ with $t \geq 2$ – a set of pre-defined completely ordered classes so that C_{h+1} is preferred to C_h, $h = 1, \ldots, t-1$; $H = \{1, 2, \ldots, t\}$.
- $A^R = \{a^*, b^*, \ldots\}$ – a set of reference alternatives which are involved in the pieces of preference information supplied by the DM. We assume that $A^R \subseteq A$.

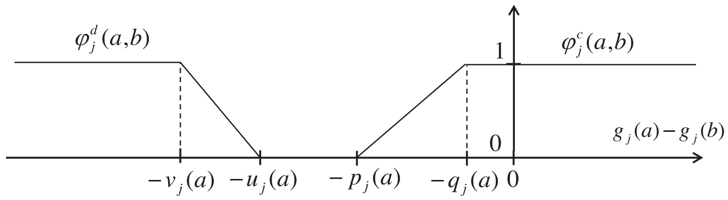

Fig. 1. The marginal concordance $\varphi_j^c(a,b)$ and discordance $\varphi_j^d(a,b)$ functions for ELECTRE.

An outranking model is one of the three main preference models used in Multiple Criteria Decision Aiding (MCDA) [21]. Its role is to represent elements of the DM's value system within the method and aggregate performances of alternatives in a way consistent with the DM's preferences. It is a general concept that can be implemented in different ways. For this purpose, the model incorporates a set of parameters with a precise semantic meaning. In this section, we recall parameters which are characteristic for the outranking models used in ELECTRE or PROMETHEE.

When instantiated with numerical values, these parameters contribute to a definition of a single outranking model instance. Applying such instance on the set of alternatives A, one can induce a preference structure in this set, called outranking relation. This relation corresponds to the statement "at least as good as". It is denoted by aSb, and its negation by aS^cb. The proper exploitation of these relations permits to arrive at the final class assignments.

The outranking preference model applies pseudo-criteria [17]. For each criterion, to indicate what performance difference is negligible and significant, we refer to the indifference ($q_j(a)$) and preference ($p_j(a)$) thresholds, respectively. These thresholds are constant in PROMETHEE, while in ELECTRE they may be defined as affine functions (e.g., $q_j(a) = \alpha_j^q g_j(a) + \beta_j^q$).

A set of weights $w_j \geq 0,\ j = 1, \ldots, m,$ is associated with the set of criteria. Without loss of generality, we assume that:

$$\sum_{j=1}^{m} w_j = 1. \tag{1}$$

2.1. ELECTRE

Verifying the truth of an outranking relation for a pair (a, b) for some outranking model instance defined in the spirit of ELECTRE [19,20] involves computation of a comprehensive concordance index $C(a, b)$, which represents the joint strength of criteria supporting aSb:

$$C(a,b) = \sum_{j=1}^{m} c_j(a,b) = \sum_{j=1}^{m} w_j \cdot \varphi_j^c(a,b), \tag{2}$$

where $\varphi_j^c(a,b)$ is the marginal concordance index defined as follows:

$$\varphi_j^c(a,b) = \begin{cases} 0 & \text{if } g_j(b) - g_j(a) \geq p_j(a), \\ 1 & \text{if } g_j(b) - g_j(a) \leq q_j(a), \\ [p_j(a) - (g_j(b) - g_j(a))]/[p_j(a) - q_j(a)] & \text{if } q_j(a) < g_j(b) - g_j(a) < p_j(a). \end{cases} \tag{3}$$

Discordance (pre-veto) $u_j(a)$ and veto thresholds $v_j(a)$ such that $v_j(a) > u_j(a) \geq p_j(a)$ are used to model the effect that a cannot outrank b if the advantage of b over a is too great on some criterion. When discordance and veto thresholds are employed, the verification of the truth of an outranking relation needs to involve computation of the non-discordance index $\Delta(a, b)$:

$$\Delta(a,b) = \prod_{j=1}^{m} (1 - \varphi_j^d(a,b)), \tag{4}$$

where $\varphi_j^d(a,b)$ is the marginal discordance index defined as follows:

$$\varphi_j^d(a,b) = \begin{cases} 0 & \text{if } g_j(b) - g_j(a) \leq u_j(a), \\ 1 & \text{if } g_j(b) - g_j(a) \geq v_j(a), \\ [(g_j(b) - g_j(a)) - u_j(a)]/[v_j(a) - u_j(a)] & \text{if } v_j(a) > g_j(b) - g_j(a) > u_j(a). \end{cases} \tag{5}$$

The marginal concordance $\varphi_j^c(a,b)$ and discordance $\varphi_j^d(a,b)$ functions are presented in Fig. 1.

Finally, let $\sigma(a, b)$ denote the credibility of comprehensive outranking of a over b:

$$\sigma(a,b) = C(a,b) \cdot \Delta(a,b). \tag{6}$$

Note that $\sigma(a, b) \in [0, 1]$, and it can be interpreted as a valued (fuzzy) outranking relation, which aggregates the results of concordance and non-discordance tests. We assume that an outranking relation aSb holds if $\sigma(a, b) \geq \lambda \in [0.5, 1]$, where λ

Table 1
Performances of two alternatives and parameters associated with three criteria used in the illustrative example.

	a	b	w_j	q_j	p_j	u_j	v_j
g_1	10	5	0.5	2	3	4	6
g_2	2	8	0.2	1	4	5	9
g_3	9	4	0.3	5	7	10	15

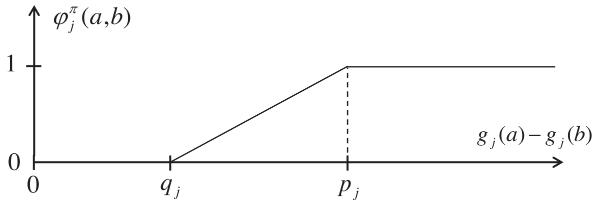

Fig. 2. The marginal preference function $\varphi_j^\pi(a, b)$ for PROMETHEE.

is a cutting level. Thus defined, S is intransitive and incomplete. Indeed, in terms of ELECTRE, aSb and bSc do not necessarily imply aSc, and for a single pair of alternatives it is possible that aS^cb and bS^ca.

Example 2.1. Let us consider a pair of alternatives a and b evaluated in terms of three gain-type criteria g_1, g_2, and g_3. The performances of alternatives and the parameters associated with the criteria are provided in Table 1.

To better illustrate the concepts of concordance and discordance indices used in ELECTRE, we show in detail how to compute the credibility of comprehensive outranking for pairs (a, b) and (b, a). For example, b is worse than a on g_3 by 5, which is, however, not more than $q_3 = 5$, and, thus, $\varphi_3^c(b, a) = 1$ and $\varphi_3^d(b, a) = 0$. Further, a is worse than b on g_2 by 6, which is more than $u_2 = 5$, but less than $v_2 = 9$. Thus, $\varphi_2^c(a, b) = 0$ and $\varphi_2^d(a, b) = (6 - 5)/(9 - 5) = 0.25$.

$$C(a, b) = w_1 \cdot \varphi_1^c(a, b) + w_2 \cdot \varphi_2^c(a, b) + w_3 \cdot \varphi_3^c(a, b) = 0.5 \cdot 1 + 0.2 \cdot 0 + 0.3 \cdot 1 = 0.8$$
$$\Delta(a, b) = (1 - \varphi_1^d(a, b)) \cdot (1 - \varphi_2^d(a, b)) \cdot (1 - \varphi_3^d(a, b)) = (1 - 0) \cdot (1 - 0.25) \cdot (1 - 0) = 0.75$$
$$\sigma(a, b) = C(a, b) \cdot \Delta(a, b) = 0.8 \cdot 0.75 = 0.6$$
$$C(b, a) = w_1 \cdot \varphi_1^c(b, a) + w_2 \cdot \varphi_2^c(b, a) + w_3 \cdot \varphi_3^c(b, a) = 0.5 \cdot 0 + 0.2 \cdot 1 + 0.3 \cdot 1 = 0.5$$
$$D(b, a) = (1 - \varphi_1^d(b, a)) \cdot (1 - \varphi_2^d(b, a)) \cdot (1 - \varphi_3^d(b, a)) = (1 - 0.5) \cdot (1 - 0) \cdot (1 - 0) = 0.5$$
$$\sigma(b, a) = C(b, a) \cdot \Delta(b, a) = 0.5 \cdot 0.5 = 0.25$$

Assuming $\lambda = 0.5$, we have $\sigma(a, b) \geq \lambda$ and $\sigma(b, a) < \lambda$. Thus, aSb and bS^ca.

2.2. PROMETHEE

When referring to the PROMETHEE methods [6], for each criterion $g_j,\ j = 1, \ldots, m$, we shall consider a marginal preference index $\varphi_j^\pi(a, b)$, such that for all $a, b \in A$:

$$\varphi_j^\pi(a, b) = \begin{cases} 0 & \text{if } g_j(a) - g_j(b) \leq q_j, \\ 1 & \text{if } g_j(a) - g_j(b) \geq p_j, \\ [(g_j(a) - g_j(b) - q_j)]/[p_j - q_j] & \text{if } q_j < g_j(a) - g_j(b) < p_j. \end{cases} \tag{7}$$

The marginal preference function $\varphi_j^\pi(a, b)$ is presented in Fig. 2.

To express the degree in which a is preferred to b over all criteria, we will refer to an aggregated (comprehensive) preference index:

$$\pi(a, b) = \sum_{j=1}^{m} w_j \cdot \varphi_j^\pi(a, b) \text{ for all } (a, b) \in A \times A. \tag{8}$$

Example 2.2. Assume the performances of alternatives a and b, the weights w_j, indifference q_j and preference p_j thresholds are the same as in Example 2.1. To better illustrate the concepts of preference indices used in PROMETHEE, we show in detail how to compute the aggregated preference indices for pairs (a, b) and (b, a). For example, a is better than b on g_1 by $5 > p_1 = 3$, and, thus, $\varphi_1^\pi(a, b) = 1$. On the contrary, the advantage of a over b on g_3 is not sufficient to warrant $\varphi_3^\pi(a, b) > 0$, because $g_3(a) - g_3(b) = 5 \leq q_3 = 5$.

$$\pi(a, b) = w_1 \cdot \varphi_1^\pi(a, b) + w_2 \cdot \varphi_2^\pi(a, b) + w_3 \cdot \varphi_3^\pi(a, b) = 0.5 \cdot 1 + 0.2 \cdot 0 + 0.3 \cdot 0 = 0.5$$
$$\pi(b, a) = w_1 \cdot \varphi_1^\pi(b, a) + w_2 \cdot \varphi_2^\pi(b, a) + w_3 \cdot \varphi_3^\pi(b, a) = 0.5 \cdot 0 + 0.2 \cdot 1 + 0.3 \cdot 0 = 0.2$$

The preference indices are further aggregated into the positive $\Phi^+(a)$ and negative $\Phi^-(a)$ outranking flows which express how much alternative a, respectively, outranks all other $n-1$ alternatives and is outranked by them:

$$\Phi^+(a) = 1/(n-1) \sum_{b \in A \setminus a} \pi(a, b) \quad \text{and} \quad \Phi^-(a) = 1/(n-1) \sum_{b \in A \setminus a} \pi(b, a). \tag{9}$$

Finally, the net outranking flow score $\Phi(a)$ reflects the difference between the positive and negative flows of a:

$$\Phi(a) = \Phi^+(a) - \Phi^-(a). \tag{10}$$

We assume that a comprehensive outranking relation aSb holds if $\Phi(a) \geq \Phi(b)$; otherwise, aS^cb. Thus defined, S is complete and transitive. Indeed, for each pair of alternatives aSb or bSa, because assigning a net outranking flow score to each alternative implies that for $a, b \in A$, $\Phi(a) \geq \Phi(b)$ or $\Phi(b) \geq \Phi(a)$. Moreover, if aSb and bSc, then $\Phi(a) \geq \Phi(b) \geq \Phi(c)$, and, thus, aSc. Finally, taking into account a specific meaning of the outranking relation, its transitivity implies that if ($a \succ b$ and bSc) or (aSb and $b \succ c$), then $a \succ c$, where $\succ$ is a preference relation such that $a \succ b$ iff aSb and bS^ca. In particular, in the context of PROMETHEE, $\Phi(a) > \Phi(b)$ and $\Phi(b) \geq \Phi(c)$ imply $\Phi(a) > \Phi(c)$.

Let us denote the set of constraints specific for each outranking model by E^{BASE}. In case of ELECTRE, these concern normalization of the weights (1), a value of the cutting level constrained to $[0.5, 1.0]$, and the way of computing a credibility of an outranking relation for each pair of alternatives (6). Note that the latter involves computation of the comprehensive (2) and marginal (3) concordances as well as comprehensive (4) and marginal (5) discordances. In case of PROMETHEE, apart from normalization of the importance coefficients (1), these constraints refer to the way of computing the net outranking flow for each alternative (10), which, in turn, incorporates the values of comprehensive (8) and marginal (7) preference indices.

The constraint set E^{BASE} defines the set of outranking model instances $\mathcal{O}^{BASE}$ in the spirit of either ELECTRE and PROMETHEE. In this perspective, each $\mathcal{O} \in \mathcal{O}^{BASE}$ can be perceived as the set of parameters instantiated with precise values whose allowed combinations are delimited by E^{BASE}.

2.3. Decision aiding process

In Fig. 3, we present the main steps of the proposed approach. The process begins by defining the problem: a set of criteria F, a set of alternatives A, their performances, and a set of ordered classes C. Then, the preference information is elicited and incorporated into the model. In case of PROMETHEE, the DM needs to provide comparison (indifference and preference) thresholds. When using ELECTRE, (s)he needs to additionally supply discordance thresholds and allowed range for the cutting level λ. Moreover, the DM may restrict a space of weights with any linear constraint. In particular, one often assumes that no criterion is more important than the remaining ones considered together, i.e., $w_j \leq 0.5$.

When it comes to the sorting-specific preference information, we assume that the DM provides:

- assignment examples specifying the desired class assignment $[C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]$ for a set of reference alternatives $a^* \in A^R$;
- assignment-based pairwise comparisons $a^* \succ^{\rightarrow}_{\geq k,DM} b^*$ and $a^* \succ^{\rightarrow}_{\leq l,DM} b^*$ involving pairs of reference alternatives $a^*, b^* \in A^R$, and specifying the minimal (k for $\succ^{\rightarrow}_{\geq k,DM}$) or maximal ($l$ for $\succ^{\rightarrow}_{\leq l,DM}$) difference of classes for each of them;
- desired class cardinalities indicating the minimal $N^{\min}_{h,DM}$ and maximal $N^{\max}_{h,DM}$ numbers of alternatives that can be assigned to class C_h.

Such indirect and imprecise preference information is used for constraining the space of outranking model instances $\mathcal{O}^{BASE}$ defined with E^{BASE}, and defining the set of outranking model instances $\mathcal{O}^R \subseteq \mathcal{O}^{BASE}$ compatible with the DM's preferences (see Section 3). This set is exploited using Linear Programming (LP) techniques to deliver three types of outcomes: possible and necessary assignments, necessary assignment-based preference relation, and extreme class cardinalities (see Section 4). Fig. 3 provides examples of different types of input preference information and conclusions that can be drawn from exploitation of the set of compatible model instances. It also clearly demonstrates the input-output correspondence.

When constructing the output recommendation, each compatible outranking model instance applies an intuitive sorting rule. The assignment of alternatives to a set of pre-defined and ordered classes is based on the following reasoning: if a outranks b, the class of a should be at least as good as the class of b. Let us provide some remarks that can be derived from this rule for the comparison of $a \in A$ with $a^* \in A^R$.

In what follows, we denote by $S_{\mathcal{O}}$ and $S^c_{\mathcal{O}}$, respectively, the outranking and non-outranking relations imposed by $\mathcal{O} \in \mathcal{O}^R$. Moreover, $L^{\mathcal{O}}(a)$ and $R^{\mathcal{O}}(a)$ are, respectively, the worst and the best class to which $\mathcal{O} \in \mathcal{O}^R$ assigns a.

Remark 2.1. If $a \in A$ outranks some reference alternative $a^* \in A^R$ for $\mathcal{O} \in \mathcal{O}^R$, then $L^{\mathcal{O}}(a)$ is not worse than $L^{DM}(a^*)$, i.e., $\forall \mathcal{O} \in \mathcal{O}^R : aS_{\mathcal{O}}a^*,\ L^{\mathcal{O}}(a) \geq L^{DM}(a^*)$. Thus, $\forall \mathcal{O} \in \mathcal{O}^R : \exists a^* \in A^R$ with $L^{DM}(a^*) = h$ and $aS_{\mathcal{O}}a^*,\ L^{\mathcal{O}}(a) \geq h$.

Remark 2.2. If $a \in A$ is outranked by some reference alternative $a^* \in A^R$ for $\mathcal{O} \in \mathcal{O}^R$, then $R^{\mathcal{O}}(a)$ is not better than $R^{DM}(a^*)$, i.e., $\forall \mathcal{O} \in \mathcal{O} : a^*S_{\mathcal{O}}a,\ R^{DM}(a^*) \geq R^{\mathcal{O}}(a)$. Thus, $\forall O \in \mathcal{O}^R : \exists a^* \in A^R$ with $R^{DM}(a^*) = h$ and $a^*S_{\mathcal{O}}a,\ R^{\mathcal{O}}(a) \leq h$.

Remark 2.3. For $\mathcal{O} \in \mathcal{O}^R$, alternative $a \in A$ is assigned to class C_h (i.e., $L^{\mathcal{O}}(a) \leq h \leq R^{\mathcal{O}}(a)$) if it does not outrank any reference alternative assigned by the DM to a class better than C_h (then, $not(L^{\mathcal{O}}(a) \geq h+1) \Rightarrow L^{\mathcal{O}}(a) \leq h$), and it is not outranked by

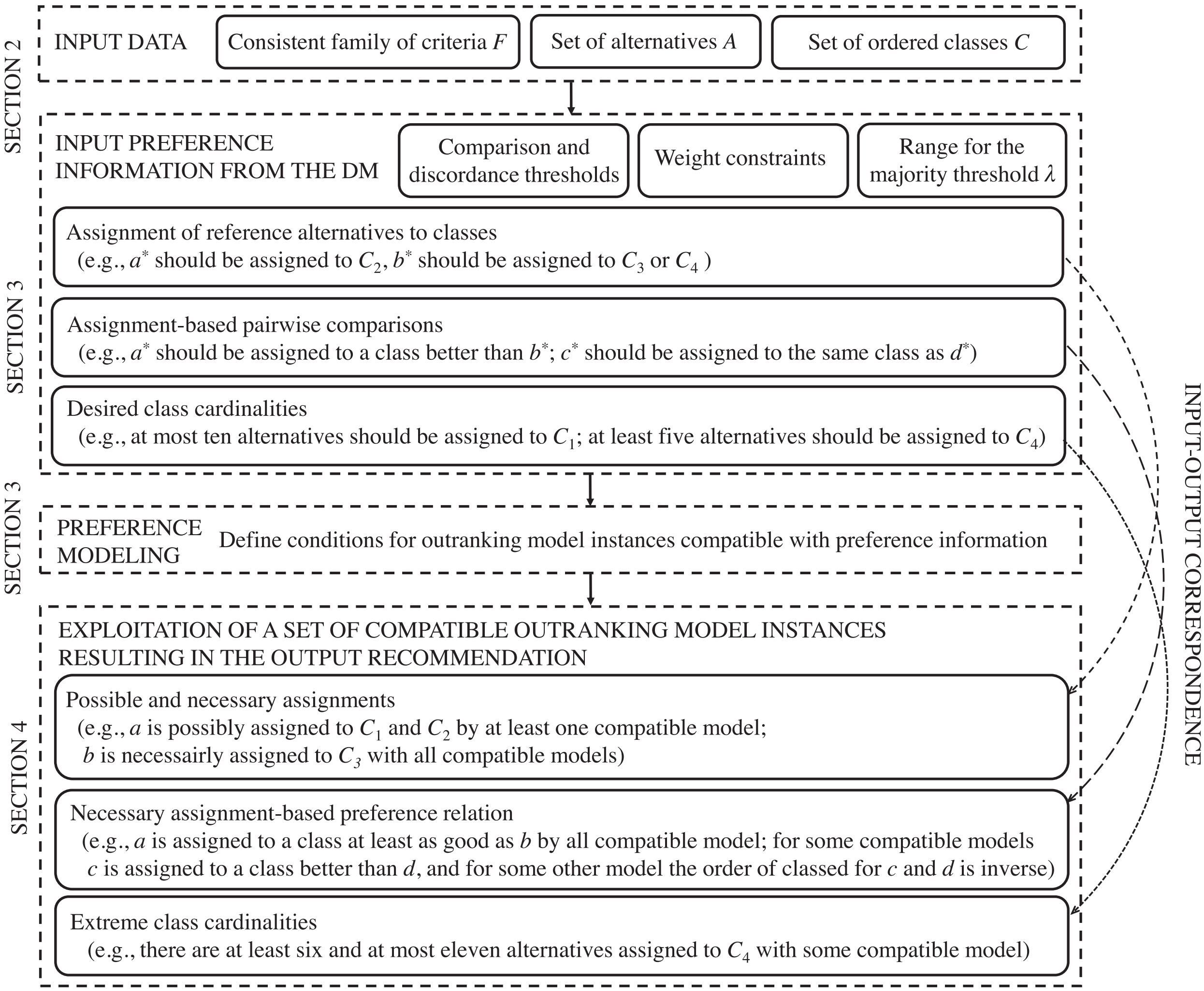

Fig. 3. Decision aiding process with the proposed approach.

Table 2
The truth of outranking relation $S_{\mathcal{O}}$ for the comparison of two alternatives a and b with four reference alternatives $a_1^* - a_4^*$ for some outranking model instance $\mathcal{O}$ used in the illustrative example.

Alt.	Rel.	Reference alternatives			
		a_1^*	a_2^*	a_3^*	a_4^*
a	(a, a_i^*)	$S_{\mathcal{O}}$	$S_{\mathcal{O}}$	$S_{\mathcal{O}}$	$S_{\mathcal{O}}^c$
	(a_i^*, a)	$S_{\mathcal{O}}^c$	$S_{\mathcal{O}}^c$	$S_{\mathcal{O}}$	$S_{\mathcal{O}}$
b	(b, a_i^*)	$S_{\mathcal{O}}$	$S_{\mathcal{O}}$	$S_{\mathcal{O}}^c$	$S_{\mathcal{O}}^c$
	(a_i^*, b)	$S_{\mathcal{O}}^c$	$S_{\mathcal{O}}^c$	$S_{\mathcal{O}}$	$S_{\mathcal{O}}$

any reference alternative assigned by the DM to a class worse than C_h (then, *not*$(R^{\mathcal{O}}(a) \le h-1) \Rightarrow h \le R^{\mathcal{O}}(a))$, i.e., $\forall \mathcal{O} \in \mathcal{O}^R$, if $\forall a^* \in A^R$ with $L^{DM}(a^*) \ge h+1$, $aS^c_{\mathcal{O}}a^*$ and $\forall a^* \in A^R$ with $R^{DM}(a^*) \le h-1$, $a^*S^c_{\mathcal{O}}a$, then $L^{\mathcal{O}}(a) \le h \le R^{\mathcal{O}}(a)$.

Example 2.3. Let us consider an outranking model instance $\mathcal{O}$ for which two alternatives a and b compare with four reference alternatives $a_1^* - a_4^*$ as indicated in Table 2. The DM defined the following assignment examples: $a_1^* \to C_1$, $a_2^* \to C_2$, $a_3^* \to C_3$, and $a_4^* \to C_4$, where C_1 and C_4 are, respectively, the worst and the best class.

According to the aforementioned sorting rule, a is assigned to C_3. Indeed, it is the sole class for which a does not outrank any reference alternative (a_4^*) assigned to a better class (C_4), and it is not outranked by any reference alternative (a_1^* and a_2^*) assigned to a worse class (C_1 or C_2). a is assigned neither to C_1 nor to C_2, because it outranks a reference alternative (a_3^*) from a better class (C_3). a is not assigned to C_4, because it is outranked by a reference alternative (a_3) from a worse class

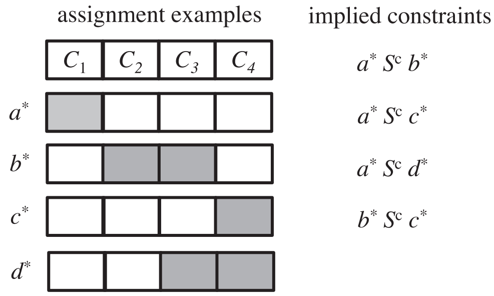

Fig. 4. Constraints implied by four exemplary assignment examples.

(C_3). Analogously, b is assigned to C_2 and C_3, because for each of the two classes the underlying assignment conditions are satisfied.

3. Preference information

In this section, we present different types of indirect preference information. We also discuss how to translate them into parameters of compatible instances of the outranking preference model.

3.1. Assignment examples

For a reference alternative $a^* \in A^R \subseteq A$, an assignment example specifies its desired class assignment $[C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]$. Its semantic meaning is that a^* is neither allowed to be assigned to a class worse than $C_{L^{DM}(a^*)}$ nor to a class better than $C_{R^{DM}(a^*)}$. In the context of outranking model, precise assignment examples with $L^{DM} = R^{DM}$ have been first considered in [43], whereas the imprecise assignments with $L^{DM} \leq R^{DM}$ have been accounted in [34].

To justify a sorting rule applied in this paper, let us remind that the truth of an outranking relation a^*Sb^* implies that a^* is either preferred or indifferent to b^*. Thus, it is reasonable to require that a class of a^* should be at least as good as a class of b^*. It is equivalent to saying that b^* is forbidden to be in a class better than a^*, if a^* is at least good as b^*. On the other hand, if $a^*S^cb^*$, b^* may be either preferred to a^*, or a^* and b^* may be incomparable. Since incomparability between a^* and b^* is admissible here and the classes are completely ordered, no relation between desired classes of a^* and b^* can be excluded. Therefore, relation S^c is inconclusive for the comparison of a^* and b^* in terms of their desired classes which are given in a preference order.

Thus, given an outranking model, a set of assignment examples is said to be consistent with it iff:

$$\forall a^*, b^* \in A^R : a^*Sb^* \;\Rightarrow\; R^{DM}(a^*) \geq L^{DM}(b^*). \tag{11}$$

The contrapositive of (11) can be formulated as:

$$\forall a^*, b^* \in A^R : R^{DM}(a^*) < L^{DM}(b^*) \Rightarrow a^*S^cb^*. \tag{12}$$

Thus, after switching the role of a^* and b^* in (12), implication (11) is equivalent to:

$$\forall a^*, b^* \in A^R : L^{DM}(a^*) > R^{DM}(b^*) \Rightarrow b^*S^ca^*. \tag{13}$$

The consistency between the assignment examples and the preference model is understood as an ability of the latter to reassign the alternatives to their desired classes [49]. The above consistency condition has been first implemented in [49] and [35].

Thus, the following constraint set E^{ASS-EX} needs to be used to translate assignment examples into parameters of compatible outranking model instances:

$$\left.\begin{array}{l}\text{for all } a^*, b^* \in A^R : L^{DM}(a^*) > R^{DM}(b^*) : \\ \;[AE_1]\; b^*S^ca^*.\end{array}\right\} E^{ASS-EX}$$

In Fig. 4, we illustrate the constraints implied by the exemplary assignments desired by the DM for four reference alternatives. The linear programs translating assignment examples into parameters of compatible instances of the preference model defined in the spirit of ELECTRE and PROMETHEE are presented in e-Appendix A.

As noted in the introduction, the role of assignment examples is to form a set of benchmarks within the set of existing alternatives defining decision classes for the possible assignment of the remaining alternatives. We assume that the DM provides assignment examples such that for each $h \in H$, $\exists a^*, b^* \in A^R, L^{DM}(a^*) = h$ and $R^{DM}(b^*) = h$. This requirement can be satisfied, e.g., when (s)he provides a precise assignment example for each class. It ensures that there exist implicit boundaries (limits) between the consecutive classes. They are subsequently referred when modeling other types of preference information or computing the outcomes of robustness analysis. However, all these procedures involve inequalities (e.g.,

$L^{DM}(c^*) \geq h+1$ or $R^{DM}(c^*) \leq h-1$) which are formulated so that when referring to the lower or upper boundary of some class, we also refer to all implicit boundaries which are, respectively, below or above. Thus, if some class boundary is not defined with the assignment examples, the proposed preference modeling remains valid, but simply the respective adjacent classes cannot be distinguished.

Note, however, that the requirement concerning the definition of implicit class frontiers through the assignment examples is not unrealistic. In fact, each multiple criteria sorting method imposes some requirements for the DM in terms of defining the classes. For example, in Electre TRI-B, (s)he needs to provide boundary class profiles; in Electre TRI-C and TRI-rC – characteristic class profiles, while in PairClas and UTADIS – assignment examples used to derive explicit class thresholds. Which of these types of required preference information is most suitable for a particular problem and DM is upon the choice of a decision analyst [51].

3.2. Desired class cardinalities

Desired class cardinality consists of a class C_h, $h \in H$, and the extreme numbers of alternatives $N^{\min}_{h,DM}$ and $N^{\max}_{h,DM}$ that can be assigned to C_h. Let us note that this type of preference information is often not related with the preference information of the DM, but rather with the context of a decision problem. Accounting for this type of preference information requires incorporating the following constraints in E^{CARD}:

- each alternative $a \in A$ will be assigned to some class, i.e., there exists some $h \in \{1, \ldots, t\}$ for which $aS^c c^*_{\geq h+1}$ with $c^*_{\geq h+1}$ being a reference alternative assigned by the DM to a class at least C_{h+1} (see $[CC_{12}]$), and $c^*_{\leq h-1} S^c a$ with $c^*_{\leq h-1}$ being a reference alternative assigned by the DM to a class at most C_{h-1} (see $[CC_{11}]$) (see Remark 2.3);
- for each class with specified cardinality constraints, the number of alternatives for which the aforementioned conditions are satisfied is within the range pre-defined by the DM (see $[CC_2]$).

$$\left.\begin{array}{l}
\text{for all } a \in A: \\
\text{for at least one } h \in H : a \rightarrow C_h, \ \text{i.e.:} \\
\left.\begin{array}{l}
[CC_{11}] \text{ if } h > 1: \ c^* S^c a, \ \text{for all } c^* \in A^R, \ \text{such that } R^{\mathrm{DM}}(c^*) \leq h-1, \\
[CC_{12}] \text{ if } h < t: \ a S^c c^*, \ \text{for all } c^* \in A^R, \ \text{such that } L^{\mathrm{DM}}(c^*) \geq h+1,
\end{array}\right\} E(a \rightarrow C_h) \\
\\
\text{for all } h \in H \text{ with specified desired class cardinality}: \\
[CC_2] \ N^{\max}_{h,DM} \geq |a \in A : a \rightarrow C_h| \geq N^{\min}_{h,DM}.
\end{array}\right\} E^{\mathrm{CARD}}$$

The MILP programs translating desired class cardinalities into parameters of compatible instances of the preference model defined in the spirit of ELECTRE and PROMETHEE are discussed in e-Appendix B. When it comes to outranking methods, desired class cardinalities have been previously considered in the context of ELECTRE TRI-B method [58], which is based on a different assignment rule than the one considered in this paper.

3.3. Assignment-based pairwise comparisons

For a pair of reference alternatives $(a^*, b^*) \in A^R \times A^R$, an assignment-based pairwise comparison specifies an imprecise valued relation between the desired assignments of a^* and b^*. We account for pairwise comparisons in the two following forms [27]:

- a^* is better than b^* by at least $k \geq 0$ classes, denoted by $a^* \succ^{\rightarrow}_{\geq k,DM} b^*$;
- a^* is better than b^* by at most $l \geq 0$ classes, denoted by $a^* \succ^{\rightarrow}_{\leq l,DM} b^*$.

When using such expressions, we do not judge a given alternative individually as in assignment examples, but rather confront alternatives one against one. In this way, we can indicate the desired assignments for pairs of alternatives, however, without specifying any concrete class. This type of preference information has not been yet considered in the context of outranking methods.

As noted in [27], providing statement $a^* \succ^{\rightarrow}_{\geq k,DM} b^*$ is equivalent to requiring that in case a^* is assigned to a class at least C_{h+k}, then b^* is assigned to a class at most C_h. To ensure that a^* is not assigned to a class worse than C_{h+k}, it needs to outrank some reference alternative assigned by the DM to a class at least C_{h+k} (see $[PCL_{11}]$). Then, the conditions for assignment of a to each class worse than C_{h+k} are not satisfied, because a outranks some reference alternative from a better class (see Remark 2.1).

If S is transitive and complete, this already guarantees that a^* is assigned to a class at least C_{h+k}. This can be explained as follows. Assume that c^*_x is a reference alternative outranked by a^* for which $L^{DM}(c^*_x) = x$, $x \in \{h+k, \ldots, t\}$ is the greatest. Then, by completeness of S, c^*_x needs to outrank all reference alternatives assigned to a class worse than C_x (thus, being preferred to them), and by transitivity of S, these reference alternatives do not outrank a^*. Since c^*_x is a reference alternative outranked by a^* for which the worst desired class L^{DM} is the best, a^* does not outrank any reference alternative for which

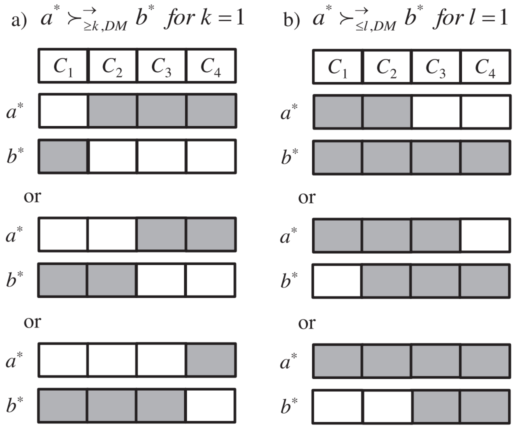

Fig. 5. Different combinations of class assignments reproducing two exemplary assignment-based pairwise comparisons: (a) $a^* \succ^{\rightarrow}_{\geq 1,DM} b^*$, (b) $a^* \succ^{\rightarrow}_{\leq 1,DM} b^*$.

$L^{DM} > x$. Consequently, all conditions justifying an assignment of a^* to C_x, $x \in \{h+k, \ldots, t\}$, are satisfied (see Remark 2.3). Thus, a^* is assigned to a class at least C_{h+k}.

If S is intransitive and incomplete, we need to ensure that the conditions for assigning a^* to any class in the range $[C_{h+k}, C_t]$ are valid (see $[PCL_{12}]$). In this case, the fact that a^* outranks some reference alternative c^*_x for which $L^{DM}(c^*_x) = x \geq h+k$ implies neither that the reference alternatives assigned to a class worse than C_x fail to outrank a^* nor that a^* fails to outrank the reference alternatives assigned to a class better than C_x. Note that this constraint is redundant if we required that each alternative needs to be assigned to some class (as explained when discussing desired class cardinalities in Section 3.2. Nevertheless, for the clarity of a separate discussion on each individual type of preference information, we leave these constraints.

Analogously, to forbid b^* being assigned to class better than C_h, it needs to be outranked by some reference alternative assigned by the DM to class at most C_h (see $[PCL_{21}]$). In case S is transitive and complete, this condition already implies that b^* is assigned to class at most C_h. If S is intransitive and incomplete, we need to additionally guarantee that the conditions for assigning b^* to any class in the range $[C_1, C_h]$ are valid (see $[PCL_{22}]$).

Since the character of this pairwise comparison is imprecise, there exist $t-k$ different combinations that need to be accounted as possible assignments for a^* and b^*, i.e., respectively, at least C_{1+k} and at most C_1, at least C_{2+k} and at most C_2, ..., at least C_t and at most C_{t-k}. In Fig. 5a, we illustrate these scenarios for $a^* \succ^{\rightarrow}_{\geq 1,DM} b^*$ for the case with four decision classes.

$$\left.\begin{array}{l}
\text{for all } a^*, b^* \in A^R : a^* \succ^{\rightarrow}_{\geq k,DM} b^* : \\
\quad \text{for at least one } h \in \{1, \ldots, t-k\} \\
[PCL_{11}] \text{ if } h+k > 1,\ a^*Sc^*, \text{ for at least one } c^* \in A^R : L^{DM}(c^*) \geq h+k, \\
[PCL_{12}] \text{ if } S \text{ is intransitive and incomplete, for at least one } i \in \{h+k, \ldots, t\} : \\
\qquad\qquad E(a^* \to C_i), \\
[PCL_{21}] \text{ if } h < t,\ c^*Sb^*, \text{ for at least one } c^* \in A^R : R^{DM}(c^*) \leq h, \\
[PCL_{22}] \text{ if } S \text{ is intransitive and incomplete, for at least one } i \in \{1, \ldots, h+k-1\} : \\
\qquad\qquad E(b^* \to C_i).
\end{array}\right\} E^{PCL}$$

Analogously, providing statement $a^* \succ^{\rightarrow}_{\leq l,DM} b^*$ is equivalent to requiring that in case a^* is assigned to class at most C_{h+l}, then b^* is assigned to class at least C_h. The justification of these requirements in terms of outranking relation is similar to the previous type of assignment-based pairwise comparison. In particular, constraints $[PCU_{11}]$ and $[PCU_{12}]$ guarantee that a^* is not assigned to a class better than C_{h+l} ($[PCU_{11}]$), but rather to some class in the range $[C_1, C_{h+l}]$ ($[PCU_{12}]$). In the same spirit, constraints $[PCU_{21}]$ and $[PCU_{22}]$ ensure that b^* is not assigned to a class worse than C_h ($[PCU_{21}]$), but rather to some class in the range $[C_h, C_t]$ ($[PCU_{22}]$). There exist $t-l$ different combinations that need to be accounted as possible assignments for a^* and b^*, i.e., respectively, at most C_{1+l} and at least C_1, at most C_{2+l} and at least C_2, ..., at most C_t and at least C_{t-l}. In Fig. 5b, we illustrate these scenarios for $a^* \succ^{\rightarrow}_{\leq 1,DM} b^*$ for the case with four decision classes.

$$\left.\begin{array}{l}
\text{for all } a^*, b^* \in A^R : a^* \succ^{\rightarrow}_{\leq l,DM} b^* : \\
\quad \text{for at least one } h \in \{1, \ldots, t-l\} \\
\quad [PCU_{11}] \text{ if } h+l < t,\ c^*Sa^*, \text{ for at least one } c^* \in A^R : R^{DM}(c^*) \leq h+l, \\
[PCU_{12}] \text{ if } S \text{ is intransitive and incomplete, for at least one } i \in \{1, \ldots, h+l\} : \\
\qquad\qquad E(a^* \to C_i), \\
[PCU_{21}] \text{ if } h > 1,\quad b^*Sc^*, \text{ for at least one } c^* \in A^R : L^{DM}(c^*) \geq h, \\
[PCU_{22}] \text{ if } S \text{ is intransitive and incomplete, for at least one } i \in \{h, \ldots, t\} : \\
\qquad\qquad E(b^* \to C_i).
\end{array}\right\} E^{PCU}$$

The MILP programs translating assignment-based pairwise comparisons of both types into parameters of compatible outranking model instances in the spirit of ELECTRE and PROMETHEE are discussed in e–Appendix C.

The set of instances of an outranking-based preference model that are compatible with preference information provided by the DM will be denoted by $\mathcal{O}^R \subseteq \mathcal{O}^{BASE}$, and the corresponding constraint set will be denoted by

$$E(A^R) = E^{BASE} \cup E^{ASS-EX} \cup E^{CARD} \cup E^{PCL} \cup E^{PCU}. \tag{14}$$

Let ε^* be the optimal value of ε obtained for the maximization of ε, subject to $E(A^R)$ (i.e., $\varepsilon^* = max\ \varepsilon$, s.t. $E(A^R)$). Note that ε is involved in the ELECTRE- and PROMETHEE-specific constraints discussed in e-Appendices A–C. If ε^* is greater than 0 while $E(A^R)$ is feasible, then there exists at least one outranking model instance compatible with the preference information.

4. Results

In this section, we present the procedures for robustness analysis investigating the stability of sorting recommendation with respect to the set of compatible outranking model instances.

4.1. Possible and necessary assignments

When checking the stability of a sorting recommendation delivered for alternative $a \in A$, we can refer to its possible assignment $C_P(a)$ ($a \rightarrow^P C_h$). It is defined as the set of indices of classes C_h for which there exists at least one compatible outranking model instance assigning a to C_h. In the context of outranking methods, the possible assignments have been previously considered in [12,34,35,49]. The possible assignment of $a \in A$ can be computed by considering Theorem 4.1.

Theorem 4.1. *$\forall a \in A$, $\forall h \in H$, $a \rightarrow^P C_h$ iff $E^{(a \rightarrow^P C_h)}$ is feasible and $\varepsilon^* = max\ \varepsilon$ s.t. $E^{(a \rightarrow^P C_h)} > 0$.*

$$\left.\begin{array}{l} E(a \rightarrow C_h), \\ E(A^R). \end{array}\right\} E^{(a \rightarrow^P C_h)}$$

Proof. To check if $h \in C_P(a)$, we assume that a is not outranked by any reference alternative assigned by the DM to a class worse than C_h, and it does not outrank any reference alternative assigned by the DM to a class better than C_h (see constraint set $E(a \rightarrow C_h)$) within a set of compatible outranking model instances $\mathcal{O}^R$ represented by $E(A^R)$. Hence $E^{(a \rightarrow^P C_h)}$ has the necessary constraints for assigning a to C_h ($a \rightarrow C_h$) for some outranking model instance $\mathcal{O} \in \mathcal{O}^R$ (see Remark 2.3). If $E^{(a \rightarrow^P C_h)}$ is feasible and $\varepsilon^* = max\ \varepsilon$ s.t. $E^{(a \rightarrow^P C_h)} > 0$, then $E(a \rightarrow C_h)$ and $E(A^R)$ are not contradictory. This, in turn, means that there exists at least one compatible outranking model instance $\mathcal{O} \in \mathcal{O}^R$ for which $a \rightarrow C_h$. Hence $h \in C_P(a)$. From another perspective, to verify whether $h \in C_P(a)$, we check if $\mathcal{O}^R$ includes some instance of the outranking preference model for which the conditions required for assigning a to C_h are satisfied. □

Remark 4.1. For $a^* \in A^R$ with $[C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]$, $C_P(a^*) \subseteq [L^{DM}(a^*), R^{DM}(a^*)]$. On the one hand, since S is reflexive (i.e., $a^* S a^*$), the conditions for assigning a^* to a class worse than $C_{L^{DM}(a^*)}$ or better than $C_{R^{DM}(a^*)}$ are not satisfied for any compatible outranking model instance $\mathcal{O} \in \mathcal{O}^R$. On the other hand, in case a desired assignment is imprecise, $C_P(a^*)$ may not include all classes from the interval $[C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]$. Firstly, a^* may outrank some other reference alternatives with the worst desired class better than $C_{L^{DM}(a^*)}$ or be outranked by some reference alternatives with the best desired class worse than $C_{R^{DM}(a^*)}$. Secondly, the possible assignment of a^* to some class in $[C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]$ may be excluded by other types of preference information such as desired class cardinalities or assignment-based pairwise comparisons.

In Section 5, we refer to the continuity of the range of possible assignments in ELECTRE methods.

The necessary assignment $C_N(a)$ is the set of indices of classes C_h for which all compatible model instances assign a to C_h ($a \rightarrow^N C_h$). When it comes to outranking methods, it has been accounted only in [34]. The necessary assignment of $a \in A$ can be computed by considering Theorem 4.2.

Theorem 4.2. *$\forall a \in A$, $\forall h \in H$, $a \rightarrow {}^N C_h$ iff $E^{(a \rightarrow^N C_h)}$ is infeasible or $\varepsilon^* = max\ \varepsilon$ s.t. $E^{(a \rightarrow^N C_h)} \leq 0$:*

$$\left.\begin{array}{l} [NA_1] \text{ if } h < t: \ aSc^* \text{ for at least one } c^* \in A^R : L^{DM}(c^*) \geq h+1, \\ \text{or} \\ [NA_2] \text{ if } h > 1: \ c^*Sa \text{ for at least one } c^* \in A^R : R^{DM}(c^*) \leq h-1, \\ E(A^R). \end{array}\right\} E^{(a \rightarrow^N C_h)}$$

Proof. To verify if $h \in C_N(a)$, we assume that a outranks some reference alternative assigned by the DM to a class better than C_h (see $[NA_1]$), or it is outranked by some reference alternative assigned by the DM to a class worse than C_h (see $[NA_2]$) within a set of compatible outranking model instances $\mathcal{O}^R$ represented by $E(A^R)$. If $[NA_1]$ is consistent with $E(A^R)$, then a is not assigned to a class worse than C_{h+1} (including C_h) for at least one compatible outranking model instance (see Remark 2.1). Analogously, if $[NA_2]$ is consistent with $E(A^R)$, then a is not assigned to a class better than C_{h-1} (including C_h) for at least one compatible outranking model instance (see Remark 2.2). Hence $E^{(a \rightarrow^N C_h)}$ has the necessary constraints for assigning a to a set of classes $[C_{L^{\mathcal{O}}(a)}, C_{R^{\mathcal{O}}(a)}]$, such that either $L^{\mathcal{O}}(a) > h$ or $R^{\mathcal{O}}(a) < h$. Let us now maximize ε subject to

$E^{(a\rightarrow^N C_h)}$. If $E^{(a\rightarrow^N C_h)}$ is infeasible or $\varepsilon^* = max\ \varepsilon$ s.t. $E^{(a\rightarrow^N C_h)} \leq 0$, then there is no outranking model instance $\mathcal{O} \in \mathcal{O}^R$ that assigns $a \in A$ to a set of classes that would exclude C_h. Hence for all compatible model instances $\mathcal{O} \in \mathcal{O}^R$: $a\rightarrow C_h$, and, thus, $h \in C_N(a)$. □

The respective LP formulations for ELECTRE and PROMETHEE are presented in e–Appendix D. Note that the robustness concern in terms of the necessary and possible assignments has been also raised in [29,32] in terms of a rule-based preference model.

4.2. Extreme class cardinalities

Theorem 4.3. *The extreme class cardinalities ($N_h^{\max}$ and $N_h^{\min}$) for class C_h can be obtained by maximizing/minimizing the number of alternatives that are simultaneously assigned to C_h, i.e., by solving the following problem:*

$$Maximize/Minimize\ |a \in A : a \rightarrow C_h|,\ \text{s.t.} \tag{15}$$

$$\left.\begin{array}{l}\text{for all } a \in A \text{ conditionally assume that } a \rightarrow C_h: \\ \quad E(a \rightarrow C_h), \\ E(A^R). \end{array}\right\} E_h^{\text{CARD}}$$

Proof. To compute the extreme class cardinalities for C_h, we assume that all alternatives are conditionally simultaneously assigned to C_h (see constraint set $E(a\rightarrow C_h)$) within a set of compatible outranking model instances $\mathcal{O}^R$ represented by $E(A^R)$. Hence E_h^{CARD} has the necessary constraints for assigning all $a \in A$ to C_h for each $\mathcal{O} \in \mathcal{O}^R$. However, these constraints are instantiated only for the minimal or maximal (depending on the formulation of an objective function) number of alternatives that can be simultaneously assigned to C_h for some $\mathcal{O} \in \mathcal{O}^R$. Due to the formulation of $E(A^R)$, the alternatives for which these constraints are not instantiated are assigned to some other class than C_h. □

Extreme class cardinalities have not been so far considered in robustness analysis of the recommendation delivered by the outranking methods. The respective MILP for ELECTRE and PROMETHEE are presented in e–Appendix E.

4.3. Necessary assignment-based preference relations

When comparing the stability of a suggested recommendation for pairs of alternatives, the necessary assignment-based preference relation $a \succsim^{\rightarrow,N} b$ holds if a is assigned to a class at least as good as class of b for all compatible outranking model instances. If a is necessarily assigned to a class at least as good as b, then the worst and the best class of a indicated by each compatible outranking model instance is not worse than, respectively, the worst and the best class of b.

The truth of necessary assignment-based preference relation for a pair $(a, b) \in A \times A$ can be verified by considering Theorem 4.4.

Theorem 4.4. *Iff $\forall h = 2, \ldots, t$, the set of constraints $E_h^{(a\succsim^{\rightarrow,N}b)}$ is infeasible or $\varepsilon^* = max\ \varepsilon$ s.t. $E_h^{(a\succsim^{\rightarrow,N}b)} \leq 0$, then $a \succsim^{\rightarrow,N} b$.*

$$\left.\begin{array}{l}[PL_1] \text{ for at least one } i \in \{1, \ldots, h-1\}: \\ \qquad E(a \rightarrow C_i), \\ \ [PL_2] \text{ for at least one } i \in \{h, \ldots, t\}: \\ \qquad E(b \rightarrow C_i), \\ \ [PL_{31}]\ bSc^* \text{ for at least one } c^* \in A^R : L^{DM}(c^*) \geq h, \\ \text{or} \\ \ [PL_{32}]\ c^*Sa \text{ for at least one } c^* \in A^R : R^{DM}(c^*) \leq h-1, \\ E(A^R). \end{array}\right\} E_h^{(a\succsim^{\rightarrow,N}b)}$$

Proof. Analogously to computation of the necessary assignment, verification of the truth of the necessary assignment-based preference relation derives from a proof by contradiction. In general, a relation (an outcome) holds for all compatible outranking model instances if the opposite relation (different outcome) is not possible even for at least one model instance. Thus, to check if $a \succsim^{\rightarrow,N} b$, we assume that the worst or the best class of b is better than, respectively, the worst or the best class of a. This can be achieved by ensuring that:

- a is assigned to some class in the interval $[C_1, C_{h-1}]$ (see $[PL_1]$), while b is placed in some class in the interval $[C_h, C_t]$ (see $[PL_2]$), and
- b is not assigned to a class worse than C_h (in this case, the worst class of b is better than the worst class of a; see $[PL_{31}]$)), or a is not assigned to a class better than C_{h-1} (in this case, the best class of b is better than the best class of a; see $[PL_{32}]$)). □

Hence $E_h^{(a \succsim^{\rightarrow,N} b)}$ has the necessary constraints for assigning b to some better class than a. If $E_h^{(a \succsim^{\rightarrow,N} b)}$ is feasible and $\varepsilon^* = max\ \varepsilon$ s.t. $E_h^{(a \succsim^{\rightarrow,N} b)} > 0$, then there exists at least one compatible outranking model instance $\mathcal{O} \in \mathcal{O}^R$ such that the worst or the best class of b is better than, respectively, the worst or the best class of a. Hence, if this is true for some $h \in \{2, \ldots, t\}$, then there exists $\mathcal{O} \in \mathcal{O}^R$, such that $L^{\mathcal{O}}(a) < L^{\mathcal{O}}(b)$ or $R^{\mathcal{O}}(a) < R^{\mathcal{O}}(b)$. On the contrary, if for all $h = 2, \ldots, t$, the set of constraints $E_h^{(a \succsim^{\rightarrow,N} b)}$ is infeasible or $\varepsilon^* = max\ \varepsilon$ s.t. $E_h^{(a \succsim^{\rightarrow,N} b)} \leq 0$, then for all $\mathcal{O} \in \mathcal{O}^R$, it holds $L^{\mathcal{O}}(a) \geq L^{\mathcal{O}}(b)$ and $R^{\mathcal{O}}(a) \geq R^{\mathcal{O}}(b)$, and, thus, $a \succsim^{\rightarrow,N} b$.

The assignment-based preference relations have not been yet considered in the context of outranking preference model. The respective MILP for ELECTRE and PROMETHEE are presented in e–Appendix F.

5. On the “no jump property” in the possible assignments for ELECTRE methods

The set $\mathcal{O}^R$ is non-convex as the space of compatible model instances is defined with a logical OR combination of linear inequalities in $E(A^R)$. This implies that there may be jumps in the possible assignment, i.e. if $\exists \mathcal{O}', \mathcal{O}'' \in \mathcal{O}^R : R^{\mathcal{O}'}(a) \leq h$ and $L^{\mathcal{O}''}(a) \geq h + \delta^h$ with $\delta^h \geq 2$, there may exist $\delta \in \{1, \ldots, \delta^h - 1\}$ for which there is no $\mathcal{O}^* \in \mathcal{O}^R$ such that $L^{\mathcal{O}^*}(a) \leq h + \delta \leq R^{\mathcal{O}^*}(a)$.

In [49], the no jump property is formulated as follows: “if a can be assigned to C_x and a can assigned in C_z $(x < z - 1)$ given $\mathcal{O}^R$ then, for all $y \in \{x+1, x+2, \ldots, z-1\}$, a can be placed in C_y”. It is proved in the following way:

On the one hand, since a can be placed in C_z, it must hold:

$$\exists \mathcal{O}' \in \mathcal{O}^R : \nexists a^* \in C_w,\ \ w < z : a^* S_{\mathcal{O}'} a. \tag{16}$$

On the other hand, since a can be placed in C_x, it must hold:

$$\exists \mathcal{O}'' \in \mathcal{O}^R : \nexists a^* \in C_b,\ \ b > x : a S_{\mathcal{O}''} a^*. \tag{17}$$

Supposing the proposition is false, a could not be placed in some C_y for $x < y < z$, i.e.:

$$\forall \mathcal{O} \in \mathcal{O}^R, [(\exists a^* \in C_{w'}, w' < y : a^* S_{\mathcal{O}} a) \vee (\exists a^* \in C_{b'}, b' > y : a S_{\mathcal{O}} a^*)]. \tag{18}$$

Thus, for every compatible outranking model instance, either (a) there exists some reference alternative a^* in a class lower than C_y that outranks a, or (b) there exists a reference alternative a^* in a class higher than C_y that is outranked by a. The authors indicate that since $w' < y < z$, then the case (a) contradicts (16), whereas since $b' > y > x$, the case (b) contradicts (17), and, thus, (18) leads to a contradiction. This is, however, incorrect. Indeed, it is based on the following reasoning:

$$\forall \mathcal{O} \in \mathcal{O}^R : (p \vee q) \Rightarrow \neg(\exists \mathcal{O}' \in \mathcal{O}^R : \neg p) \wedge \neg(\exists \mathcal{O}'' \in \mathcal{O}^R : \neg q). \tag{19}$$

The above implication is false, because for $\mathcal{O}' \in \mathcal{O}^R$, q may be true, or for $\mathcal{O}'' \in \mathcal{O}^R$, p may be true. Thus, the no jump property neither holds for the approach presented in [49].

6. Illustrative study

In this section, we illustrate the use of both ELECTRE- and PROMETHEE-specific integrated sorting frameworks. Let us consider real-world data concerning 23 Polish research units dealing with bioinformatics (group of joint evaluation called NZ1B) to be assigned to one of the four classes C_1–C_4 (with C_1 being the worst, and C_4 the best one). The units are evaluated on the following four criteria with an interval scale and an increasing direction of preference:

- scientific activity (g_1; an average number of points gained by a research unit member for her/his scientific publication in the international journals and for patents),
- scientific potential (g_2; reflects the unit's ability to grant scientific degrees, number of scientific titles granted in the evaluation period, and prestigious memberships of researchers),
- material effects of unit's activities (g_3; money acquired from grants and industry),
- remaining effects of unit's activities (g_4; ten most important achievements of unit's members evaluated by experts of the Ministry).

For a detailed description of the criteria, see [31]. The data we analyze are derived from the report of Polish Ministry of Science and Higher Education concerning the last parameterization of research units conducted in Poland in 2013[1]. The performances of 23 considered research units (denoted by RU1-RU23) are given in Table 3.

We assume that there is no dictatorial criterion, and that the impact of each criterion cannot be fully neglected. Thus, we use the following constraints with respect to the criteria weights: $0.01 \leq w_j \leq 0.5$. The indifference and preference thresholds provided by the DM are given in Table 4. We use the same threshold values for ELECTRE and PROMETHEE. Note, however, that these thresholds are used to define comparison functions which have slightly different interpretation (for details, see [28]). To illustrate another difference between these methods, we account for a veto phenomenon when using ELECTRE. In particular, we define the discordance and veto thresholds for g_1 (see Table 4).

[1] http://www.nauka.gov.pl/g2/oryginal/2013_09/485ab765cf1189945f7b95572d728cb0.pdf.

Table 3
Performances of research units.

	g_1	g_2	g_3	g_4
RU1	87.98	457	8.25	90.00
RU2	80.78	707	12.39	67.48
RU3	78.59	470	6.70	95.00
RU4	78.52	969	6.12	47.79
RU5	74.17	1085	6.29	65.00
RU6	77.29	1061	2.95	75.00
RU7	72.00	657	4.97	90.00
RU8	70.21	125	35.25	40.00
RU9	72.16	705	2.14	65.00
RU10	72.95	540	1.77	65.00
RU11	66.82	477	6.47	60.00
RU12	61.02	516	10.59	65.00
RU13	62.56	529	5.23	75.00
RU14	60.25	472	5.82	65.00
RU15	63.08	421	1.60	60.00
RU16	51.47	441	2.52	50.00
RU17	40.36	268	16.3	55.00
RU18	42.56	279	4.01	60.00
RU19	48.17	47	1.20	20.00
RU20	45.18	264	0.76	40.00
RU21	41.76	359	0.32	40.00
RU22	36.23	505	0.37	42.00
RU23	39.48	382	0.57	30.00

Table 4
Indifference, preference, discordance, and veto thresholds provided by the Decision Maker (for PROMETHEE, only the indifference and preference thresholds are used).

	q_j	p_j	u_j	v_j
g_1	2	4	10	15
g_2	20	40	–	–
g_3	1	2	–	–
g_4	2	4	–	–

Table 5
Indirect preference information.

Assignment examples	Assignment-based comparisons	Class cardinalities
RU2 $\rightarrow C_4$	$RU3 \succ^{\rightarrow}_{\geq 0} RU14$	C_4 [2–11]
RU4 $\rightarrow C_3$	$RU11 \succ^{\rightarrow}_{\geq 0} RU18$	C_3 [2–11]
RU12 $\rightarrow C_3$	$RU5 \succ^{\rightarrow}_{\geq 1} RU19$	C_2 [2–11]
RU15 $\rightarrow C_2$		C_1 [2–11]
RU20 $\rightarrow C_2$		
RU21 $\rightarrow C_1$		

When it comes to the holistic judgments, we assume the DM to have provided six precise assignment examples, three assignment-based pairwise comparisons, and imprecise desired class cardinalities for each class. These are given in Table 5. When it comes to assignment-based pairwise comparisons, we require that RU3 and RU11 are assigned to a class at least as good as RU14 and RU18, respectively, while RU5 should be assigned to a class better than RU19. As for the desired class cardinalities, each class should accommodate at least two units, but not more than 50% of all units.

The possible and necessary assignments obtained with ELECTRE and PROMETHEE are presented in Table 6 (columns C_P and C_N). The inferred compatible outranking model instances reproduce all assignment examples, i.e., RU2 is assigned to C_4, RU4 to C_3, etc. When it comes to ELECTRE, for 5 non-reference units the possible assignment is precise, 11 units are possibly assigned to two consecutive classes, and only RU11 is possibly placed in any class between C_2 and C_4. The greatest number of alternatives is assigned to C_3–C_4. They always outrank some reference alternative assigned by the DM to C_3, which prevents them from units assigned to class worse than C_3. For all units but RU3 the necessary assignments are non-empty. In particular, there are 2 units necessarily assigned to C_4, 10 to C_3, 5 to C_2, and 3 to C_1. For example, the necessary assignment to C_3 means that the respective units neither outrank nor are outranked by any other reference unit assigned by the DM to, respectively, C_4 or C_2. For RU11 and RU17, the necessary assignments are not univocal, whereas for RU3 there is no agreement between all compatible outranking model instances with respect to its assignment, and, thus, $C_N(\text{RU3}) = \emptyset$.

Table 6
Possible (C_P) and necessary (C_N) assignments obtained with ELECTRE and PROMETHEE.

	ELECTRE		PROMETHEE	
	C_P	C_N	C_P	C_N
RU1	C_4	C_4	C_4	C_4
RU2	C_4	C_4	C_4	C_4
RU3	C_3–C_4	–	C_3–C_4	C_4
RU4	C_3	C_3	C_3	C_3
RU5	C_3–C_4	C_3	C_3–C_4	C_3
RU6	C_3–C_4	C_3	C_2–C_4	–
RU7	C_3–C_4	C_3	C_2–C_4	–
RU8	C_2–C_3	C_3	C_2–C_4	–
RU9	C_3–C_4	C_3	C_2–C_4	C_3
RU10	C_3–C_4	C_3	C_2–C_4	C_3
RU11	C_2–C_4	C_2–C_3	C_2–C_3	C_3
RU12	C_3	C_3	C_3	C_3
RU13	C_3–C_4	C_3	C_2–C_4	C_3
RU14	C_3	C_3	C_2–C_3	C_3
RU15	C_2	C_2	C_2	C_2
RU16	C_2	C_2	C_2	C_2
RU17	C_1–C_2	C_1–C_2	C_2–C_3	–
RU18	C_1–C_2	C_2	C_2–C_3	C_2
RU19	C_1–C_2	C_2	C_1–C_2	–
RU20	C_2	C_2	C_2	C_2
RU21	C_1	C_1	C_1	C_1
RU22	C_1	C_1	C_1–C_2	–
RU23	C_1	C_1	C_1	C_1

When it comes to PROMETHEE, for only 3 non-reference units the possible assignment is precise. When compared with ELECTRE, C_P is the same for 12 units, while being more or less precise for, respectively, 1 and 8 units. For RU17 and RU18 the possible assignments are different though the intersection with the results provided by ELECTRE is non-empty. The non-empty necessary assignments for 17 units agree with those obtained previously. Since C_N is empty for 6 units, the recommendations suggested by all compatible outranking model instances should be regarded as more conflicting than in case of ELECTRE.

The observed extreme class cardinalities respect desired cardinalities provided by the DM, being, however, even more constrained for most decision classes. For example:

- for ELECTRE there are at least 4, 7, 11, and 2 units assigned to the respective classes between C_1 and C_4; note that the sum of these minimal cardinalities exceeds the number of alternatives $n = 23$, because the individual compatible ELECTRE model instances assign some alternatives imprecisely to a set of decision classes; e.g., RU11 and RU17 are necessarily assigned to two consecutive classes (C_2–C_3 and C_1–C_2, respectively), thus, increasing the cardinality of both these classes for each compatible model instance;
- for PROMETHEE there are at most 4, 11, 11, and 9 units placed in, respectively, C_1, C_2, C_3, and C_4.

It is interesting to collate these class cardinalities with the possible assignments. For example, in case of PROMETHEE there are 4, 15, 14, and 10 units possibly assigned to the preference-ordered classes. Thus, for all classes but C_1 the number of units which are simultaneously assigned to it by some compatible outranking model instance is less than the number of units which are, in general, assigned to this class by at least one model instance.

The graphs of the necessary assignment-based preference relation obtained with both methods are presented in Fig. 6. Obviously, they reproduce all assignment-based pairwise comparisons. Thus, RU3$\succ^{\rightarrow}_{\geq 0}$RU14, RU11$\succ^{\rightarrow}_{\geq 0}$RU18, and RU5$\succ^{\rightarrow}_{\geq 1}$RU19 imply the truth of $\succsim^{\rightarrow,N}$ for these pairs. Moreover, this relation holds also for pairs of units assigned by the DM to different classes. For example, RU2 (assigned by the DM to C_4) is necessarily preferred to RU4 (assigned to C_3).

For both ELECTRE and PROMETHEE, we can observe three types of relations derived from $\succsim^{\rightarrow,N}$. Let us provide their intuitive explanation, referring to the exemplary outcomes of PROMETHEE:

- RU14 is connected by an arc with RU11 in Fig. 6b, thus, being necessarily strictly preferred to RU11 in terms of the assignment-based relation. This means that RU11 is assigned to a class at least as good as RU14, while the opposite statement is no true. Thus, RU14 is assigned to a class worse than RU11 with some compatible outranking model instances even though the possible assignments for this pair of units are the same.
- RU15 and RU16 are grouped together within a single node in Fig. 6b, thus, being indifferent in terms of the necessary assignment-based relation. This means that RU15 and RU16 are assigned to the same decision classes with all compatible outranking model instances.
- RU8 and RU9 are not connected by an arc in Fig. 6b, thus, being incomparable in terms of the necessary assignment-based relation. This means that with some compatible outranking model instances RU8 is assigned to a class better than RU9, while with some other model instances, the order of classes is inverse.

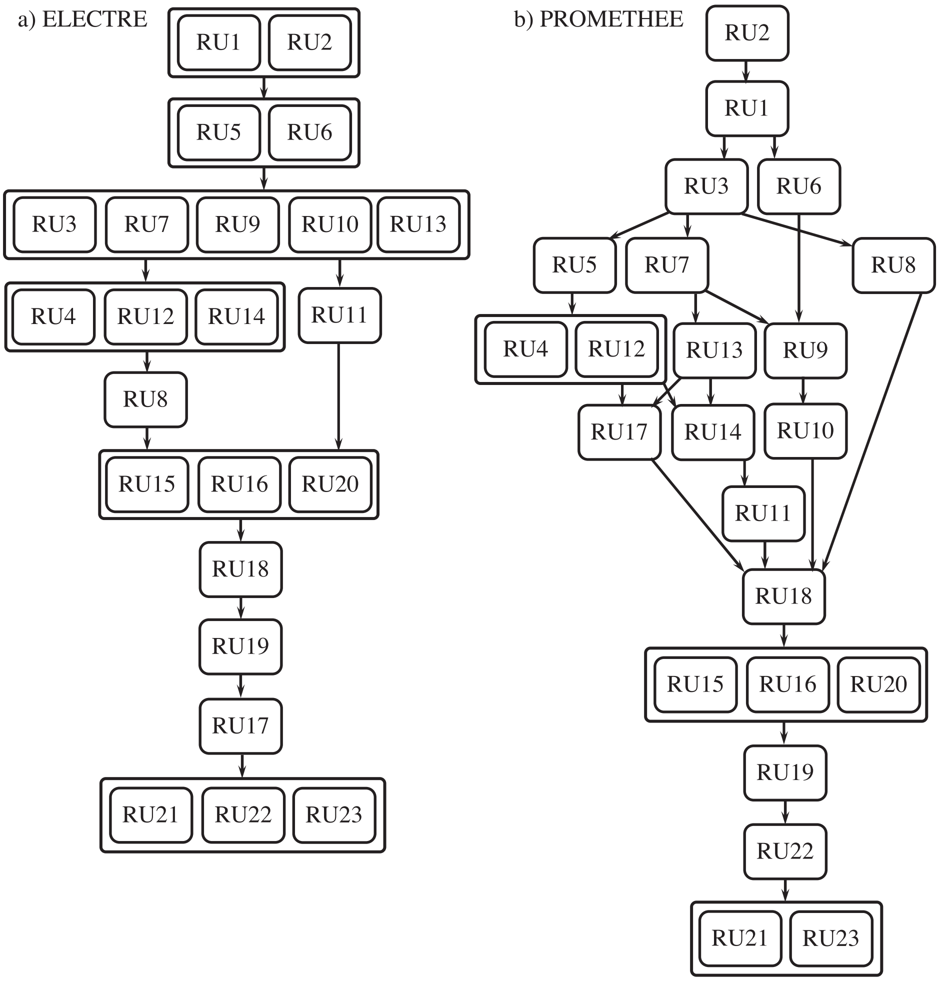

Fig. 6. Diagrams of $\succsim^{\rightarrow,N}$ for (a) ELECTRE, and (b) PROMETHEE.

7. Preference Modeling and Robustness Analysis for Model Instances Providing Precise Assignments

In the previous sections, we presented the methods which used outranking models with implicit class boundaries defined by the assignment examples. This implied that a single preference model instance may have assigned each alternative to a set of decision classes. In this case, the mathematical programs dealing with the assignment-based pairwise comparisons and desired class cardinalities at the input as well as the assignment-based preference relations and extreme class cardinalities at the output involve numerous binary variables. As a result, although being more general and avoiding introduction of arbitrary explicit class boundaries, these models may be considered efficiently with the contemporary MILP solvers only in case of relatively low dimensional problems.

In this section, we prove how preference modeling can be simplified in case a single preference model instance provides precise assignments for the alternatives. As an example of such model we consider a threshold-based sorting procedure with alternatives judged in terms of their comprehensive scores, $Sc(a)$ for $a \in A$. These scores can be e.g.:

- comprehensive values derived from an additive value function $U = \sum_{j=1}^{m} u_j(a)$ as in the UTADIS method; in this case, aSb if $U(a) \geq U(b)$;
- net outranking flows as in the PROMETHEE-like PairClas method [14]; in this case, aSb if $\Phi(a) \geq \Phi(b)$.

The threshold-based procedure assumes that vector $\mathbf{b} = \{b_0, b_1, \ldots, b_{t-1}, b_t\}$ is defined so that $0 = b_0 < b_1 < \ldots < b_{t-1} < 1 < b_t$, and b_{h-1} and b_h are the boundary thresholds of class C_h, $h = 1, \ldots, t$. Thus, a is assigned to C_h iff $b_{h-1} \leq Sc(a) < b_h$.

Example 7.1. Let us consider a preference model instance for which the scores of two alternatives c and d compare with the scores of eight reference alternatives $a_1^* - a_8^*$ as indicated in Fig. 7. The DM defined the following assignment examples: $a_1^*, a_2^* \rightarrow C_1$, $a_3^*, a_4^* \rightarrow C_2$, $a_5^*, a_6^* \rightarrow C_3$, and $a_7^*, a_8^* \rightarrow C_4$. This allowed the method to infer the class thresholds $b_1 - b_3$ for the explicit separation of classes. According to the threshold-based sorting rule, c and d are assigned to, respectively, C_2 and C_3.

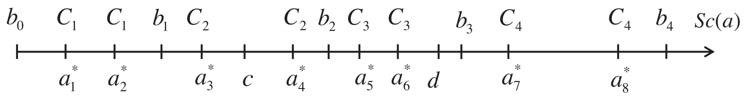

Fig. 7. Threshold-based sorting procedure involving two alternatives c and d and eight reference alternatives $a_1^* - a_8^*$.

When a preference model instance delivers precise assignments, for each alternative $a \in A$ a verified assignment to C_h excludes its assignment to all other classes than C_h. For example, if $b_{h-1} \leq Sc(a) < b_h$, then a is assigned to C_h, but at the same time the conditions for its assignment to C_l, $l \in H \backslash \{h\}$, are not satisfied. On the contrary, such phenomenon does not occur when class boundaries are defined implicitly by the assignment examples. For example, if a is not outranked by any reference alternative assigned to a class worse than C_h and it does not outrank any reference alternative assigned to a class better than C_h, the conditions for assignment of a to C_h are verified. However, their truth does not exclude the assignment of a to some other class than C_h. Consequently, for methods using preference model instances which deliver precise assignments, a single binary variable $v(a, h)$ is sufficient to control the conditions which guarantee an assignment of a to C_h while excluding all other assignments. This is not the case for the approaches recommending possibly imprecise assignments (e.g., the variants of ELECTRE and PROMETHE presented in the previous sections).

In case of a threshold-based sorting procedure, this can be modeled in the following way:

$$\left.\begin{array}{l} \text{for all } a \in A, h \in H: \\ \quad M(1 - v(a,h)) + Sc(a) \geq b_{h-1}, \\ \quad M(v(a,h) - 1) + Sc(a) + \varepsilon \leq b_h, \\ \text{for all } a \in A: \\ \quad \sum_{h=1}^{t} v(a,h) = 1, \\ b_0 = 0, b_t = 1 + \varepsilon, \\ \text{for } h = 1, \ldots, t: \\ \quad b_{h-1} + \varepsilon \leq b_h, \\ \text{for all } a \in A, \ h = 1, \ldots, t: \\ \quad v(a,h) \in \{0, 1\}, \end{array}\right\} E_{VAR}^{BIN}$$

where M and ε are, respectively, arbitrarily large and small positive values. Indeed, if $v(a, h) = 1$, then $b_{h-1} \leq Sc(a) < b_h$, and, thus, a is assigned precisely to C_h. When $v(a, h) = 0$, a is assigned to a class other than C_h.

To demonstrate how to incorporate different types of preference information and conduct robustness analysis, let us denote the basic set of constraints by $E_{BIN-VAR}^{BASE}$. For UTADIS it would be composed of E_{VAR}^{BIN} with $Sc(a) = U(a)$, monotonicity and normalization constraints for the additive value function (see [22]). For PairClas it is composed of E_{VAR}^{BIN} with $Sc(a) = \Phi(a)$, (1), and (10). Nevertheless, the mathematical programs presented in the following sections are independent of the underlying model as long as it provides precise assignments. Let us emphasize that when applied in the context of value-based UTADIS method, these formulations are more concise than the proposal described in [27].

To better explain the underlying ideas, we will provide an example on how to model each type of preference information or how to check the truth of some result while referring to our illustrative study. Thus, we will use the binary variables $v($ RUi, $h)$ for $i = 1, \ldots, 23$, and $h = 1, 2, 3, 4$.

7.1. Preference information

Assignment examples. To ensure that a range of allowed assignments for reference alternative $a^* \in A^R$ is limited to $[C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]$, we need to require that $v(a^*, h) = 1$ (meaning that a^* is assigned to C_h) for $h \in \{L^{DM}(a^*), \ldots, R^{DM}(a^*)\}$, i.e.:

$$\left.\begin{array}{l} \text{for all } a^* \rightarrow [C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]: \\ \quad [AE^P] \ \sum_{h=L^{DM}(a^*)}^{R^{DM}(a^*)} v(a^*, h) = 1. \end{array}\right\} E_{BIN-VAR}^{ASS-EX}$$

When taking into account that a^* can be assigned to a single class only, i.e., $\sum_{h=1}^{t} v(a^*, h) = 1$, this already implies that $v(a^*, h) = 0$ for $h \in [1, \ldots, L^{DM}(a^*)) \cup (R^{DM}(a^*), \ldots, t]$. For example, the assignment of RU4 to C_3, which is desired by the DM in our illustrative study, is guaranteed with $v(\text{RU4}, 3) = 1$.

Desired class cardinalities. Since $v(a, h) = 1$ implies that a is assigned to C_h, $\sum_{a \in A} v(a, h)$ corresponds to the cardinality of C_h. Thus, to respect desired class cardinalities it is sufficient to impose constraints on the sum of binary variables associated with a particular class, i.e.:

$$\left.\begin{array}{l} \text{for each } C_h, \ h \in H: \\ \quad [CL^P] \ \sum_{a \in A} v(a,h) \geq N_{h,DM}^{\min}, \\ \quad [CU^P] \ \sum_{a \in A} v(a,h) \leq N_{h,DM}^{\max}. \end{array}\right\} E_{BIN-VAR}^{CC}$$

For example, the class cardinality desired by the DM for C_2 translates into $2 \leq \sum_{i=1}^{23} v(\text{ RU}i, 2) \leq 11$.

Assignment-based pairwise comparisons. To guarantee that $a^* \succ^{\rightarrow}_{\geq k,DM} b^*$, a^* and b^* need to be assigned to a class at least C_{h+k} and at most C_h, respectively, for $h \in \{1, \ldots, t-k\}$. Thus, the difference between indices of classes to which a^* and b^* are assigned needs to be not less than k, i.e.:

$$\left.\begin{array}{l}[PCL^P] \text{ for all } a^*, b^* \in A^R : a^* \succ^{\rightarrow}_{\geq k,DM} b^* : \\ \sum_{h=1}^{t} h \cdot v(a^*, h) \geq [\sum_{h=1}^{t} h \cdot v(b^*, h)] + k.\end{array}\right\} E^{PCL}_{BIN-VAR}$$

Note that in the above formulation only the indices corresponding to $v(a^*, h) = 1$ and $v(b^*, h) = 1$ are multiplied by one, thus, being instantiated, whereas the remaining indices, being multiplied by zero, are neglected. For example, the assignment-based pairwise comparison RU5 $\succ^{\rightarrow}_{\geq 1}$ RU19 is modeled as follows:

$$\begin{aligned}&1 \cdot v(\text{RU5}, 1) + 2 \cdot v(\text{RU5}, 2) + 3 \cdot v(\text{RU5}, 3) + 4 \cdot v(\text{RU5}, 4)\\ &\quad \geq 1 \cdot v(\text{RU19}, 1) + 2 \cdot v(\text{RU19}, 2) + 3 \cdot v(\text{RU19}, 3) + 4 \cdot v(\text{RU19}, 19) + 1.\end{aligned}$$

The above inequality is satisfied only if $v(\text{RU5}, h) = 1$ for some $h \in \{2, 3, 4\}$ and $v(\text{RU19}, l) = 1$ for $l = 1$, or $v(\text{RU5}, h) = 1$ for some $h \in \{3, 4\}$ and $v(\text{RU19}, l) = 1$ for some $l \in \{1, 2\}$, or $v(\text{RU5}, h) = 1$ for $h = 4$ and $v(\text{RU19}, l) = 1$ for some $l \in \{1, 2, 3\}$. Each of these three scenarios implies that RU5 is assigned to a class better than RU19.

Further, if a^* and b^* should be assigned to the same class, this can be modeled as follows:

$$\left.\begin{array}{l}[PCE^P] \text{ for all } a^*, b^* \in A^R : a^* \sim^{\rightarrow}_{DM} b^* : \\ \text{for } h = 1, \ldots, t : \ v(a^*, h) = v(b^*, h).\end{array}\right\} E^{PCE}_{BIN-VAR}$$

Thus, the binary variables $v(a^*, h)$ and $v(b^*, h)$ need to be instantiated with one for the same $h \in H$, and equal to zero for the remaining class indices.

The mathematical model $E^{PCU}_{BIN-VAR}$ for $a^* \succ^{\rightarrow}_{\leq l,DM} b^*$ can be formulated analogously to $E^{PCL}_{BIN-VAR}$, i.e.:

$$\left.\begin{array}{l}[PCU^P] \text{ for all } a^*, b^* \in A^R : a^* \succ^{\rightarrow}_{\leq l,DM} b^* : \\ \sum_{h=1}^{t} h \cdot v(a^*, h) \leq [\sum_{h=1}^{t} h \cdot v(b^*, h)] + l.\end{array}\right\} E^{PCU}_{BIN-VAR}$$

The index of a class b^* is assigned to ($v(b^*, h) = 1$) can be by at most l greater than the index of a class a^* is assigned to ($v(a^*, h) = 1$).

Let us denote the set of constraints defining outranking model instances which are compatible with different types of preference information by $E^{MODEL}_{BIN-VAR}$. It is composed of $E^{BASE}_{BIN-VAR}$, $E^{ASS-EX}_{BIN-VAR}$, $E^{CC}_{BIN-VAR}$, $E^{PCL}_{BIN-VAR}$, and $E^{PCU}_{BIN-VAR}$.

7.2. Robustness analysis

Possible assignment. To check if a is possibly assigned to C_h, it is sufficient to verify if $v(a, h) = 1$ does not contradict $E^{MODEL}_{BIN-VAR}$ (see $E^{(a \rightarrow^P C_h)}_{BIN-VAR}$). If so, there exists at least one compatible preference model instance assigning a to C_h, and, thus $h \in C_P(a)$.

$$\left.\begin{array}{l}[P_1] \ v(a, h) = 1, \\ E^{MODEL}_{BIN-VAR}.\end{array}\right\} E^{(a \rightarrow^P C_h)}_{BIN-VAR}$$

For example, when checking if RU5 is possibly assigned to C_3, $[P_1]$ would have the following form: $v(\text{ RU5}, 3) = 1$.

Necessary assignment. To verify if a is necessarily assigned to C_h, we need to check if it can be possibly assigned to any other class by assuming $v(a, h) = 0$ (see $E^{(a \rightarrow^N C_h)}_{BIN-VAR}$). If $v(a, h) = 0$ contradicts $E^{MODEL}_{BIN-VAR}$, then $h \in C_N(a)$, because it is not possible that a is assigned to some other class than C_h.

$$\left.\begin{array}{l}[N_1] \ v(a, h) = 0, \\ E^{MODEL}_{BIN-VAR}.\end{array}\right\} E^{(a \rightarrow^N C_h)}_{BIN-VAR}$$

For example, when checking if RU15 is necessarily assigned to C_4, $[N_1]$ would have the following form: $v(\text{ RU15}, 4) = 0$.

Possible assignment-based preference relation. To check if a is possibly assigned to a class at least as good as b, it is sufficient to verify if assuming $v(a, h_a) = 1$ and $v(b, h_b) = 1$ for $h_a \geq h_b$ is consistent with $E^{MODEL}_{BIN-VAR}$ (see $E^{(a \succsim^{P,\rightarrow} b)}_{BIN-VAR}$). Note that $[AP_1]$ guarantees that a class of a is at least as good as a class of b by suitably comparing the indices of classes these alternatives are assigned to. If so, $a \succsim^{P,\rightarrow} b$.

$$\left.\begin{array}{l}[AP_1] \ \sum_{h=1}^{t} h \cdot v(a, h) \geq \sum_{h=1}^{t} h \cdot v(b, h), \\ E^{MODEL}_{BIN-VAR}.\end{array}\right\} E^{(a \succsim^{P,\rightarrow} b)}_{BIN-VAR}$$

For example, when checking if RU7 is possibly assigned to a class at least as good as RU8, $[AP_1]$ would have the following form:

$$\begin{aligned}&1 \cdot v(\text{RU7}, 1) + 2 \cdot v(\text{RU7}, 2) + 3 \cdot v(\text{RU7}, 3) + 4 \cdot v(\text{RU7}, 4)\\ &\quad \geq 1 \cdot v(\text{RU8}, 1) + 2 \cdot v(\text{RU8}, 2) + 3 \cdot v(\text{RU8}, 3) + 4 \cdot v(\text{RU8}, 4).\end{aligned}$$

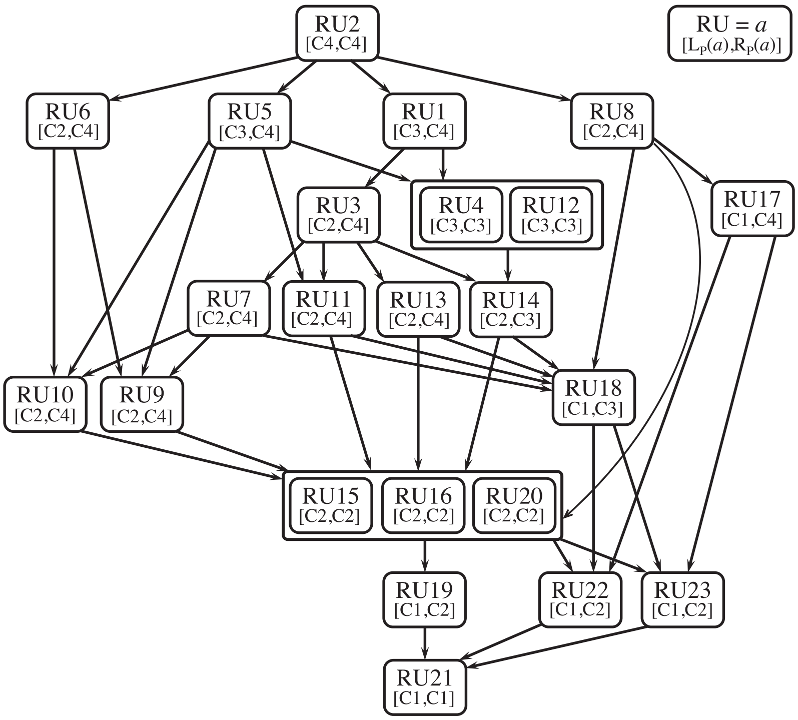

Fig. 8. Diagram of $\succsim^{\rightarrow,N}$ and possible assignments for UTADIS.

Necessary assignment-based preference relation. To verify if a is necessarily assigned to a class at least as good as b, we need to prove that b cannot be assigned to a class strictly better than a for at least one compatible model instance. Note that $[AN_1]$ guarantees that a class of b is better than a class of a by suitably comparing the indices of classes these alternatives are assigned to. If $v(b,h_b)=1$ and $v(a,h_a)=1$ considered together for $h_b > h_a$ contradict $E^{MODEL}_{BIN-VAR}$ (see $E^{(a\succsim^{N,\rightarrow}b)}_{BIN-VAR}$), then $a \succsim^{N,\rightarrow} b$.

$$\left.\begin{array}{l}[AN_1]\ \sum_{h=1}^{t} h\cdot v(b,h) \geq \sum_{h=1}^{t} h\cdot v(a,h)+1,\\ E^{MODEL}_{BIN-VAR}.\end{array}\right\} E^{(a\succsim^{N,\rightarrow}b)}_{BIN-VAR}$$

For example, when checking if RU5 is necessarily assigned to a class at least as good as RU4, $[AN_1]$ would have the following form:

$$1\cdot v(\text{RU4},1)+2\cdot v(\text{RU4},2)+3\cdot v(\text{RU4},3)+4\cdot v(\text{RU4},4)$$
$$\geq 1\cdot v(\text{RU5},1)+2\cdot v(\text{RU5},2)+3\cdot v(\text{RU5},3)+4\cdot v(\text{RU5},4)+1.$$

Extreme class cardinalities. The maximal $N_h^{\max}$ and minimal $N_h^{\min}$ cardinality of class C_h can be derived by optimizing $\sum_{a\in A} v(a,h)$ in the set of compatible preference model instances, i.e.:

$$\text{Maximize / Minimize}: \sum_{a\in A} v(a,h), \ \text{s.t.}\ E^{MODEL}_{BIN-VAR}. \tag{20}$$

For example, when computing the extreme class cardinalities of C_1, we would optimize $\sum_{i=1}^{23} v(\text{RUi},1)$.

7.3. Results for the illustrative study for preference model delivering precise assignments

In this section, we report the results obtained for our study regarding classification of research units using an exemplary method which delivers precise assignments. For illustrative purpose, we apply a value-based UTADIS method using a preference modeling framework described in the previous subsections. The input preference information in form of assignment examples, assignment-based pairwise comparisons, and desired class cardinalities remains the same as in Section 6.

The main differences when comparing UTADIS with ELECTRE and PROMETHEE consist in neglecting the use of discriminating thresholds and criteria weights as well as in the possible compensation between alternatives' gains and losses due to an additive aggregation of marginal values. These characteristics imply that the value-based preference model of UTADIS is more flexible and, thus, the diversity of obtained results is richer than in case of ELECTRE or PROMETHEE. In Fig. 8, we present the graph of the necessary assignment-based preference relation along with the possible assignments for all alternatives. In general, when compared to the results obtained in Section 6:

- the possible and necessary assignments are less precise; e.g., RU1 is now possibly assigned to $[C_3, C_4]$ and its necessary assignment is empty, while it was possibly and necessarily assigned only to C_4 with both ELECTRE and PROMETHEE; note that for preference models whose instances deliver precise assignments, the necessary assignment is non-empty only if the possible assignment is precise (for our study, this holds for RU2, RU3, RU12, RU15, RU16, RU20, and RU21);
- there are more incomparabilities in the necessary assignment-based preference relations (these pairs are not connected by an arc in Fig. 8; e.g., RU1 and RU5 are now incomparable in terms of $\succsim^{\rightarrow, N}$, because for some model instances RU1 is assigned to C_4, while RU5 is placed in C_3, while for some other model instances the order of classes is inverse; on the contrary, when using ELECTRE and PROMETHEE, RU1 has been always assigned to a class at least as good as RU5;
- the observed extreme class cardinalities are less precise ($C_1 - [2, 6]$, $C_2 - [3, 11]$, $C_3 - [2, 11]$, and $C_4 - [2, 10]$).

8. Conclusions

In this paper, we introduced an integrated framework for preference modeling and robustness analysis in outranking-based multiple criteria sorting. We ensured a correspondence between input preference information and output recommendation in terms of three perspectives: assignment of individual alternatives, size of decision classes, and comparison between assignments of pairs of alternatives. We discussed how to translate different types of preference information into parameters of compatible outranking model instance and how to derive various kinds of sorting results. First, we referred only to a semantic meaning of an outranking relation. Then, we discussed two specific implementations of our proposal with an outranking model defined in the spirit of either ELECTRE or PROMETHEE. Finally, we showed how the complexity of mathematical preference modeling and robustness analysis can be reduced when considering a set of preference model instances providing precise assignments for the alternatives. For illustrative purpose, we referred to the PairClas and UTADIS methods, but the latter proposal can be used with any outranking- or value-based model delivering precise assignments.

We envisage the adaptation of the proposed framework to outranking-based models with boundary or characteristic class profiles [1,34,57] and to approaches which construct the results being most consistent with the provided preference information (see, e.g., the THESEUS method [15]) in case all holistic judgments provided by the DM cannot be reproduced together with at least one compatible outranking model instance. Other appealing directions of future research include accounting for a hierarchical structure of criteria [53,54] as well as considering fuzzy performance values [8,9,44] instead of deterministic ones.

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Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.ins.2016.02.059.

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